How much do you know about geography?
What is the capital of Serbia?
What is the capital of Serbia?

Belgrade
1. General Place Description

2. Distancing Perception & Spatial Connectivity

3. Geographical Imaginations

From Anne-Laure Freant
Geodesy, Coordinate Systems, and Map Projections
Objectives

• Introduce the main concepts of geodesy – coordinate systems, datums, ellipsoids, and geoids
• Describe map projections and coordinate systems we use in GIS
• Two key questions – what is the shape of the earth, and where are things on the earth?
Our first references were local, relative, and qualitative.
Even today, addresses are qualitative locations ASSIGNED by a government body, but how do we know the quantitative location?
..with a coordinate “address,” also assigned by governments ....but, how is Lat/Lon defined, and or height, and what is this WGS84 business?
Coordinates Differ Because:

1) Our best estimates of where things are have evolved over time (pre- and post-satellite technologies (Geodesy))

2) We use different methods to “project” our data from the curved Earth surface to a flat map surface (Map Projections, next lecture)
Geodesy: Why all the confusion?
All measurements are relative to some reference, and our best estimates of the reference have changed (better measurements, better models).

We started with, visualize the World, and still use 2-dimensional systems of coordinates. This won’t work for long distances, or the whole Earth.

The Earth’s shape is approximately spherical, but irregular. We didn’t know this at first, and we’ve since discovered relative positions are changing (plate tectonics).
Describe, as quantitatively and precisely as possible, where A and B are.
For most the reaction is to:

• Establish an origin (starting reference, or reference frame)
• Identify, through measurements, the locations of important “things” using this reference frame
• All positions are relative to the origin and principal directions within this reference frame
Cartesian Coordinates (right angle)

Two dimensions, X and Y, or E and N

Two-dimensional system most often used with Projected coordinates (maps and GIS)
• Establish a reference frame
• Identify, through measurements, the locations of important “things” using this reference frame
Geodesy - science of measuring the size and shape of the Earth

Datum - a reference surface
e.g., a site datum - a reference height against which elevations are measured

![Diagram of datum heights](image-url)
Datum Example
Except for the Earth, our standard locations were

first, local “fixed” objects (large rock outcrops, placed monuments,

then, estimated imaginary lines on the surface (Greenwich meridian, the Equator)

then, the center of mass of the Earth, and standard axes
Datum Evolution

Pre 1500’s

1500’s to 1980s

post 1980s
Two Kinds of Surveying

- **Plane Surveying**
  > We can assume the underlying Earth (our reference surface) is flat, with topographic changes over this flat base
  > Only valid over “short distances”

- **Geodetic Surveying**
  > Takes Earth’s curvature into account - our reference surface is curved
  > But the calculations become more complicated
Plane Surveying is Based on Horizontal and Vertical Distance Measurement

Horizontal measurements required “leveling”

- either raising the end of the measuring rods to maintain a level measurement, or

- measuring angles and calculating the horizontal distance
Surveying Methods Evolved Through Time

1600s

1700s

1800s
1600’s - scales and optics
1800's - early 1900's, improved precision, but same methods, over much larger scope and distances
Earth Curvature Distortion

S is the surface distance
C is a “straight-line” distance
R is the radius (about 6,357 km for Earth)
θ is an angle

What is the difference between S and C (in meters)?
# How Big a Difference?

<table>
<thead>
<tr>
<th>Angle, $\theta$</th>
<th>Curved Distance, $S$</th>
<th>Straight line Distance, $C$</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 second</td>
<td>30.8195 m</td>
<td>30.8195 m</td>
<td>0.0000 m</td>
</tr>
<tr>
<td>1 minute</td>
<td>1,894.5 m</td>
<td>1,894.5</td>
<td>0.0000 m</td>
</tr>
<tr>
<td>30 minutes = 0.5 degree</td>
<td>55.475 km</td>
<td>55.475</td>
<td>0.1760 m</td>
</tr>
<tr>
<td>1 degree</td>
<td>110.95 km</td>
<td>110.95 km</td>
<td>1.408 m</td>
</tr>
<tr>
<td>5 degrees</td>
<td>554.75 km</td>
<td>554.75 km</td>
<td>176.0 m</td>
</tr>
</tbody>
</table>
At some point, we have to move from a plane to a globe.

What is our reference “surface” for a globe?

It depends

When we thought the World was a sphere, we invented a spherical system, with spherical references
Horizontal location on a sphere, using Spherical Coordinates

- Use angles of rotation to define a directional vector
- Use the length of a vector originating near the ellipsoid center to define the location on the surface
Measuring Earth Size, Shape, and Location of Points is Complicated

The Earth Isn’t Round

The Continents are Moving

The Poles Wander
Eratosthenes deduced the Earth’s circumference to within a few percent of our present estimate.
Earth’s is Flattened - an Ellipsoid

Two radii:
- $r_1$, along semi-major (through Equator)
- $r_2$, along semi-minor (through poles)
Contradictory Measurements, Cleared Up by the French

A French expedition measured the same angle at high and low latitudes.
If arc 1 = arc 2, Earth a sphere
If arc 1 < arc 2, Earth flattened ellipsoid in the equatorial plane
If arc 1 > arc 2, Earth flattened at poles

measurements showed arc 1 > arc 2
No one could agree on the right size
- Differening estimates of $r_1$ and $r_2$?

- Different countries adopted different “standard” ellipsoids
Different Countries Adopted Different Ellipsoids as Datums

Why?

• Surveys rarely spanned continents

• Best fit differed

• Nationalism
Local or Regional Ellipsoid

Origin, $R_1$, and $R_2$ of ellipsoid specified such that separation between ellipsoid and Geoid is small.

These ellipsoids have names, e.g., Clarke 1880, or Bessel.
Global Ellipsoid

Selected so that these have the best fit “globally”, to sets of measurements taken across the globe.

Generally have less appealing names, e.g. WGS84, or ITRF 2000
<table>
<thead>
<tr>
<th>Name</th>
<th>Year</th>
<th>Equatorial Radius, $r_1$ meters</th>
<th>Polar Radius, $r_2$ meters</th>
<th>Flattening Factor $f$</th>
<th>Users</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airy</td>
<td>1830</td>
<td>6,377,563.4</td>
<td>6,356,256.9</td>
<td>1/299.32</td>
<td>Great Britain</td>
</tr>
<tr>
<td>Bessel</td>
<td>1841</td>
<td>6,377,397.2</td>
<td>6,356,079.0</td>
<td>1/299.15</td>
<td>Central Europe, Chile, Indonesia</td>
</tr>
<tr>
<td>Clarke</td>
<td>1866</td>
<td>6,378,206.4</td>
<td>6,356,583.8</td>
<td>1/294.98</td>
<td>Most of Africa; France</td>
</tr>
<tr>
<td>Clarke</td>
<td>1880</td>
<td>6,378,249.1</td>
<td>6,356,514.9</td>
<td>1/293.46</td>
<td>North America; Philippines</td>
</tr>
<tr>
<td>Intern-</td>
<td>1924</td>
<td>6,378,388.0</td>
<td>6,356,911.9</td>
<td>1/297.00</td>
<td>Much of the World</td>
</tr>
<tr>
<td>au-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>austr-</td>
<td>1965</td>
<td>6,378,160.0</td>
<td>6,356,774.7</td>
<td>1/298.25</td>
<td>Australia</td>
</tr>
<tr>
<td>WGS72</td>
<td>1972</td>
<td>6,378,135.0</td>
<td>6,356,750.5</td>
<td>1/298.26</td>
<td>NASA, U.S. Dept. of Defense</td>
</tr>
<tr>
<td>GRS80</td>
<td>1980</td>
<td>6,378,137.0</td>
<td>6,356,752.3</td>
<td>1/298.26</td>
<td>Worldwide</td>
</tr>
</tbody>
</table>
Different Ellipsoids in Different Countries

There are locally "best fit" ellipsoids
Transforming Between Reference Surfaces

We can transform positions from one ellipsoid to another via mathematical operations e.g., an origin shift, and mapping from one surface to the next.
Geocentric Ellipsoid and Coordinate System

- 3-D Cartesian system
- Origin (0,0,0) at the Earth center of mass
- Best globally fit spheroid, e.g., WGS84, used for global coordinate systems
a) convert source datum geographic to Cartesian

b) shift origin (translation)

c) scale

d) rotate

e) convert target datum Cartesian to geographic
The Earth’s True Shape a GEOID

Geoid

Ellipsoid
Why?

Law of Gravity:

\[ \text{Force} = g \times \text{Mass}_a \times \text{mass}_b \]

\[ \frac{r^2}{r^2} \]

Larger mass, stronger pull, and the Earth's mass isn't uniform throughout, e.g., near mountains, subsurface ore, deep mantle density variation.
Definition of the Geoid

A Geoid is the surface perpendicular to a plumb line, and for which the pull of gravity is a given value.

Note, there are at least three definitions of vertical:
• A plumb direction
• The center of the celestial disk
• Perpendicular to the adopted ellipsoid
The Geoid is a measured surface

Gravimeter

CIRES, U. of Colorado, and Gerd Steinle-Neumann, U. Bayerisches
The Geoid is a measured surface

Gravimeter on a Satellite
Separation between the Geoid and best-fitting global ellipsoid averages about 30 meters – this “undulation” is always below 100 meters.
We have three surfaces to keep track of at each point on Earth:

1. the ellipsoid
2. the geoid, and
3. the physical surface
“Horizontal” Position

Our position on the surface of the Earth is defined by a latitude / longitude pair on a specified ellipsoid (also known as a spheroid)
Heights are usually specified relative to the Geoid

Heights above the geoid are *orthometric heights*. These are the heights usually reported on topographic or other maps.

Heights above the ellipsoid are spheroidal heights. Sometimes used to define vertical position, but usually to specify geoid-ellipsoid separation.
ellipsoidal height = orthometric height + geoidal height

\[ h = H + N \]
Defining a Datum

Horizontal Datum
- Specify the ellipsoid
- Specify the coordinate locations of features on this ellipsoidal surface

Vertical Datum
- Specify the ellipsoid
- Specify the Geoid – which set of measurements will you use, or which model
Specifying a Horizontal Datum

- Measure positions (celestial observations, surveys, satellite tracking)
- Adjust measurements to account for geoid, determine position on adopted ellipsoid
Specifying a Horizontal Datum

A horizontal datum is a reference ellipsoid, plus a precisely-measured set of points that establish locations on the ellipsoid.

These points define the reference surface against which all horizontal positions are measured.
Defining the Horizontal Datum

Horizontal datums are realized through large survey networks.

1. Define the shape of the Earth (the ellipsoid)
2. Define the location of a set of known points – control points – for which the position on the ellipsoid is precisely known
3. This is the reference surface and network against which all other points will be measured
Datum, Survey Network

Historically:
- Triangulation Network
- Astronomical observation
- Intermittent baselines
- Multiple, redundant angle measurements

Why these technologies?
- Easy to measure angles
- Difficult to measure distance accurately
- Time consuming to measure point position accurately
Astronomical Observations Were Accurate, but Time-Consuming

\[ \theta = \gamma - \alpha \]

radius = \( \frac{a}{\theta} \)
Horizontal Surface Measurements
Horizontal Surface Measurements: Compensation Bars
Triangulation

If we measure the initial baseline length $A$, and measure the angles $a$, $b$, and $c$, we are then able to calculate the lengths $B$ and $C$:

by the law of sines, \[ \frac{A}{B} = \frac{\sin(a)}{\sin(b)} \]

then \[ B = A \frac{\sin(b)}{\sin(a)} \] and \[ C = A \frac{\sin(c)}{\sin(a)} \]
Triangulation

With the length $C$ known, angles $e, f,$ and $g$ may then be measured. The law of sines may be used with the now known distance $C$ to calculate lengths $E$ and $F$. Successive datum points may be established to extend the network using primarily angle measurements.
Horizontal Survey Benchmarks
Survey Network, 1900

Figure 4.1. U.S. horizontal control network in 1900. (from Schwartz, 1989)
North American Datum of 1927, 26,000 measured points, Clarke 1866 spheroid, fixed starting point in Kansas

NAD27 survey network
Survey Network, 1981

(from Schwartz, 1989)
NAD83
North American Datum 1983

Successor to NAD27.

250,000 measurements points
GRS80 Ellipsoid,
No fixed stations

(from Schwartz, 1989)
North American Datum of 1927, 26,000 measured points, Clarke 1866 spheroid, fixed starting point in Kansas
NAD83
North American Datum 1983

Successor to NAD27.

250,000 measurements points
GRS80 Ellipsoid,
No fixed stations

(from Schwartz, 1989)
Datum “Adjustment”

A datum adjustment is a calculation of the coordinates of each benchmark – this is how we specify the “reference surface”

Not straightforward, because of:

Errors in surface measurements from 800 BC to 1940 AD

After 1960, errors in assumptions came to dominate
Two Main Classes of Datums

Pre-satellite: e.g., Clarke, Bessel, NAD27, NAD83(1986)
• large errors (10s to 100s of meters),
• local to continental

Post-satellite: e.g., NAD83(HARN), NAD83(CORS96), WGS84(1132), ITRF99
• small relative errors (cm to 1 m)
• global
### Examples of Datum Shifts

New Jersey control point, successive datum transformations applied

<table>
<thead>
<tr>
<th>Datum</th>
<th>Longitude (W)</th>
<th>Latitude (N)</th>
<th>Shift (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAD27</td>
<td>74° 12' 3.86927&quot;</td>
<td>40° 47' 0.76531&quot;</td>
<td>36.3</td>
</tr>
<tr>
<td>NAD83 (1986)</td>
<td>74° 12' 2.39240&quot;</td>
<td>40° 47' 1.12762&quot;</td>
<td>0.04</td>
</tr>
<tr>
<td>NAD83 (HARN)</td>
<td>74° 12' 2.39069&quot;</td>
<td>40° 47' 1.12762&quot;</td>
<td>0.05</td>
</tr>
<tr>
<td>NAD83 (CORS96)</td>
<td>74° 12' 2.39009&quot;</td>
<td>40° 47' 1.12936&quot;</td>
<td>0.05</td>
</tr>
<tr>
<td>WGS84 (G1150)</td>
<td>74° 12' 2.39720&quot;</td>
<td>40° 47' 1.15946&quot;</td>
<td>0.95</td>
</tr>
</tbody>
</table>

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[Diagram showing locations and shifts]
There are Three Main Families of Datums
a) convert source datum geographic to Cartesian
b) shift origin (translation)
c) scale
d) rotate
e) convert target datum Cartesian to geographic
NAD27
NAD27 and NAD83
Vertical Datums

Like horizontal, but referenced to standard elevation and established using vertical leveling

Two major vertical datums,

North American Vertical Datum of 1927 (NAVD29), and an update,

North American Vertical Datum of 1988 (NAVD88)
Summary

• The Earth’s shape is approximated by an ellipsoid
• Horizontal positions defined relative to the ellipsoid
• A datum is a reference surface, a realization of the ellipsoid, against which locations are measured
• Multiple datums, improved through time
Summary

• Ellipsoid is an approximation – the shape is better characterized by a geoid
• Heights measured relative to the geoid
• Estimates of geoid changes through time
• Sets of horizontal, vertical points and reference surfaces are datums
Cartesian Coordinates (right angle)

Two dimensions, X and Y, or in three dimensions, the X, Y, and Z

Two-dimensional system most often used with *Projected coordinates*

Three dimensional system used with *Geocentric coordinates*
Spherical Coordinates

- Use angles of rotation to define a directional vector
- Use the length of a vector originating near the ellipsoid center to define the location on the surface
longitude

latitude

meridians

north pole

Greenwich meridian

parallels

equator

Greenwich

meridian

90°

north pole

90°

south pole

-90°

-67.5°

-45°

-22.5°

0°

-90°

-45°

-67.5°

22.5°

45°

67.5°

-45°

-67.5°

22.5°

45°

67.5°